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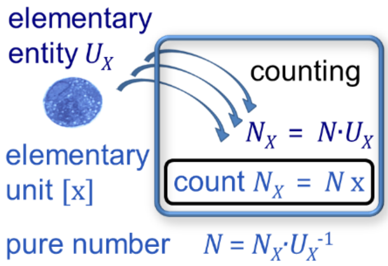

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The elementary unit — count and numbers in the International System of Units

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Summary

The International System of Units (SI) defines counting quantities with unit one, symbol 1. Conventionally, the unit symbol 1 is not shown explicitly. This leads to ambiguities in expressing quantities per count and implies an insufficient distinction between *count* and *number*. Building on a critical reading of the SI Brochure (9th edition, version 3.01, August 2024) and formal analysis of counting, the Euclidean Unit U_X is recognized as the individual entity: a single item of, e.g., an event, atom, molecule, particle of type X), distinguished from the abstract *elementary unit*. The symbol ‘1’ is replaced by ‘x’, representing the abstract unit of U_X and count N_X.

A count N_X with unit x is then defined as $N_X = N \cdot U_X$, where U_X with unit x represents a single elementary entity of type X and N is a dimensionless number. This notation ensures consistency between units of extensive quantities, such as cell count ($N_{ce} = 2 \cdot 10^6$ x = 2 Mx), and quantities normalized per count. The inverse Avogadro constant N_A^{-1} [mol/x] and elementary charge e [C/x] link the count to amount and charge, respectively. Consequently, the units of the fundamental constants are distinct from the units of the elementary quantities amount [mol] and charge [C].

A consistent nomenclature resolves persistent terminological confusion between numbers, units, and entities. The symbol ‘x’ instead of ‘1’ aligns the unit of count with the principles of quantity calculus. Treating x like all symbols of SI units improves dimensional consistency, supports accurate interpretation of physical quantities, and enhances the rigor of communication across the natural sciences.

Numbers cannot be seen, heard, touched, tasted, or smelled; they do not emit or reflect signals; they leave no traces. So what kind of things are they? These are the central philosophical questions about numbers. Plausible combinations of answers have proved elusive. — Marcus Giaquinto (2015)

1. Introduction

"There are also some quantities that cannot be described in terms of the seven base quantities of the SI, but are quantities that are a number of entities. Examples are a number of cellular or biomolecular entities, or degeneracy in quantum mechanics. These quantities are also quantities with the unit one" ([1] p. 136). ".. values of quantities with the unit one can be expressed simply as numbers. The unit symbol 1 or unit name "one" are rarely explicitly written" ([1] p. 151).

The International System of Units (SI) defines physical quantities in terms of seven base units and several derived units, establishing a globally consistent foundation for science and technology, trade and commerce. Among these, the unit one (symbol 1) for 'a number of entities' occupies a unique position, yet a general definition of the quantity *count* is missing. Unlike other SI units, the unit one may be omitted in expressions, which has significant implications [2-5, 13]. When quantities are expressed per count — such as energy per particle or mass per cell — the one or 1 disappears from the dimensional analysis, leading to ambiguity [10]. When a quantity is expressed in kilograms [kg], is it the mass of the total system, or the mass per elementary entity? Without an explicit symbol for the unit of count, such a distinction remains obscured. The SI Brochure (9th edition) [1] acknowledges this practice but does not resolve the underlying conceptual issues.

In a formal analysis of counting, before counting begins, an *(entetic) Euclidean Unit* U_X is defined in terms of a countable item belonging to elementary entity-type X (e.g., an event, atom, molecule or particle of type X). The number of elementary entity-types and thus units U_X is practically unlimited in nature with its quantum structure and in the imaginary world of theoretically countable items. An elementary entity U_X is distinguished from the count N_X, but both are expressed in the same abstract *elementary unit*. The elementary unit is a unit comparable with all other abstract SI units, such as the mole for amount, the coulomb for charge, or the kilogram for mass. For consistency with the nomenclature of SI units, the symbol for the elementary unit must not be a numeral. "Roman (upright) type, in general lower-case, is used for symbols of units" ([1] p. 162). The SI symbol '1' for the unit of counting quantities is not consistent with this SI Resolution 7 [1]. Symbol 'x' is suggested instead, avoiding the confounding of the symbol for an abstract unit with symbols for entities [4, 13]. The proposed notation clarifies the link between the count and other elementary quantities, particularly in the expression of fundamental constants such as the elementary charge

e [C/x] and the Avogadro constant N_A [x/mol]. Intriguingly, the defining constants e and N_A have the units [C] and [mol^{-1}] in the SI ([1] p. 128).

Grounding abstract units in seven defining constants of the SI marks the revolutionary departure from the previous reliance on material artefacts ([1] p. 122). However, there remains a fundamental ambiguity to be resolved in defining the unit of the quantity *count* in the SI. The introduction of elementary entity U_X and elementary unit x eliminates semantic ambiguity and enhances dimensional consistency. It brings the treatment of the SI unit one into alignment with the general rules of quantity calculus and improves clarity in fields ranging from statistical mechanics and chemical stoichiometry to systems biology and metrology. By addressing the conceptual status of the quantity count and its unit, this contribution aims to strengthen the foundation of the SI and improve the interpretability of physical quantities across the sciences.

2. Before counting begins

Count is one of the most fundamental quantities in the natural and social sciences. Closely connected to the concept of numbers [8], a count is perceived rather intuitively, is present universally according to the quantum structure of nature, and its "unit one is the neutral element of any system of units – necessary and present automatically. There is no requirement to introduce it formally by decision" ([1] p. 136). Formally, however, counting involves multiple steps (Figure 1). Before counting can begin, the item being counted must be specified. This foundational concept is often implicit but deserves explicit treatment. An item relates to something in the real or imagined world, either a dynamic event that can be perceived repeatedly over time, or a static material object that can be repeatedly discerned from other ocurrences. An item (from the Latin adverb *item* 'likewise, moreover, in addition') refers to an elementary entity to which one of the same type of things can be added. Thus a 'proto-item' serves as a reference: this 'elementary entity' is the *entetic* Euclidean Unit representing the type of event or object as 'a single individual thing' in Euclid's *Elements, Book VII* [8].

Formalization requires the creation of a symbol coherent with the system of symbols for quantities. The *entetic* or Euclidean Unit U_X defines the type X of the individual thing — a single body or a single event of type X. U_X is the symbol indicating a single countable elementary entity of entity-type X (the specified thing) with numerical value one. If X is 'cells' ce, then U_{ce} is the unit cell. Thus the Euclidean Unit is *realized* in the real world, and conceptually *realized* as a formal quantity in a formal system of quantities and units, in the cognitive sequence of counting.

Abstract units u_Q are *abstracted* from the nature of the entetic Euclidean Unit U_X. A single unit of something is a quantity (Q_X = 1 U_X) and not an abstract unit (u_Q = x) (Figure 2). In contrast to units u_Q, it is essential to qualify an Euclidean Unit U_X by X to define the 'single individual thing': the unit particle, the unit atom O, the unit O_2 molecule, the unit endothelial cell, the unit organism of type org, the unit event of type

X. The term "nature of the quantity" is ambiguous, with dualistic meaning as (*1*) the nature of the quantity Q_X being mass, length, volume, amount of substance, etc.; or (*2*) the nature of entity X, e.g., neutrons versus protons, O_2 versus CO_2, endothelial versus neuronal cells. Did the SI Brochure intend the second interpretation, or both?

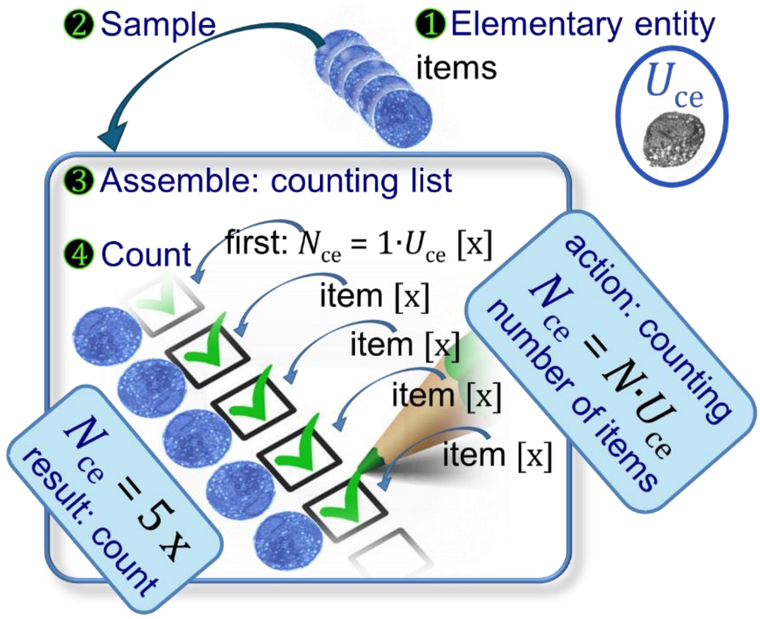

Figure 1. Counting. The elementary entity cell, U_{ce}, is the *material* Euclidean Unit of the cell count N_{ce}, expressed in the *abstract* elementary unit [x] with the meaning 'one item'. Cells are a subset of entities X which are countable objects. A sample of cells can be expressed as a *count*, the number of single individual items. The single individual cell defines the elementary U_{ce}. A count $N_X = N{\cdot}U_X$ equals the number of U_X. X must represent the same entity in both occurrences. An elementary entity U_X is not a count (U_X is not a number of U_X). Both quantities N_X and U_X have the same abstract unit, the 'elementary unit' [x]. The cell count N_{ce} is an elementary quantity obtained by counting the number N of cells U_{ce}, item after item. Modified after Gnaiger 2020 [9].

In the context "amount of substance n_S(X) of any pure sample S of entity X" ([1] p. 210), the term entity and symbol X are used with the meaning *elementary entity-type* X. Since not all entity-types X are countable, the term 'elementary' accounts for the class of things that are suitable for enumeration [2]. In contrast, when referring to "the number of entities in one mole of any substance" ([1] p. 210), the term entities is understood as the number of *elementary entities* U_X. The quantity U_X, however, is not a count N_X, but its recognition is a prerequisite for initiating the counting process with N_X = 1 U_X (Figure 1).

3. Sampling and assembling

> "For example, the equation $p = 48$ kPa may equally be written as $p/\text{kPa} = 48$. It is common practice to write the quotient of a quantity and a unit in this way for a column heading in a table, so that the entries in the table are simply numbers" ([1] p. 148).

Sampling is required to collect items for transfer to the 'counting table' (Figure 1). This may be the pipetting of cells onto a microscope slide to be used for cell counting.

Real items may be placed upon a physical table; symbols of items may be placed into a list — a virtual table of rows and columns. Are numerical entries in a table numbers or numerals (symbols for numbers)? In any table, items must not only be collected but properly arranged. A counting scheme is prepared for *assembling* items on the counting table. The microscope slide is fixed on the microscope stage for counting the items; a team of football players is positioned in line to count the members; a numeral is oriented horizontally to count the digits; numerals are organized vertically in a column, such that the number of rows with numerals can be counted (Excel has a specific mathematical function resulting in this count).

4. The count and symbol x

Finally, *counting* entails adding sequentially item after item into the counting list yielding the count $N_X = N{\cdot}U_X$, where U_X denotes a single elementary entity of type X (e.g., molecule, cell, body, but also event), and N is a pure number. When a counting process of items B ($X \stackrel{\text{def}}{=} \mathrm{B}$) is completed and no more items B are added, the resulting count is $N_\mathrm{B} = N\,\mathrm{x}$ (Figure 1). The symbol x is not a new unit, but a notational aid to make the unit of count explicit and distinguishable. The SI unit 'one' is the 'elementary unit', where 'one' (an) and 'unit' (uno) express the same concept. The quantity N_B then denotes a count of items B, expressed in the elementary unit with symbol x, which has the intended typographical similarity with the mathematical multiplication symbol $\times$ (times).

Count is the number of items (likewise entities). A count $N_\mathrm{ce} = 5\ \mathrm{x}$ can be spelled out as 'the count of cells is five times' (Figure 1). In common language, a cell count concentration N_ce/V of $1{\cdot}10^6\ \mathrm{x/mL}$ is said to be 'one million cells per milliliter' or $1{\cdot}10^6$ ce/mL. This divides, however, a real thing (cells) by an abstract unit $[\mathrm{mL}]$. The SI gives the count a unit of a number with numeral 1 as its symbol. In case of an explicit notation of this symbol [4], the above cell count concentration would be written as $1{\cdot}10^6\ 1/\mathrm{mL}$, evidently a good reason to omit the symbol 1. The inconsistency is solved by introducing the symbol x for the abstract elementary unit of any count, which is also the abstract unit of the entetic unit U_X. Then a cell count concentration of $1{\cdot}10^6\ \mathrm{x/mL}$ can be expressed as $1\ \mathrm{Mx/mL}$, using the prefix mega for 10^6.

"Whilst not part of the SI, descriptive terms are commonly used in place of the unit one to indicate the entities being considered when expressing the values of quantities that are a number of entities" ([1] p. 151). The number of different types of specified events and types of countable objects U_X is practically unbounded, as is the vast number of entity types that can be expressed in the unit mole $[\mathrm{mol}]$. In contrast, there is a single abstract unit either for the count $[\mathrm{x}]$ or amount $[\mathrm{mol}]$. The *abstract unit* of a count is the elementary unit $[\mathrm{x}]$ – applicable to all kinds of count. The unit x does not specify if an event is simultaneous, random or periodic in time, or if the entity oxygen is atomic O or molecular O_2. Consistent with the fact that it "is important to give a precise definition of the entity involved" ([1] p. 134) for amount of substance (with unit 'mole'), the specific entity of a count must be defined separately from its abstract unit x. Count concentration C_X (with the ambiguous IUPAC term 'number concentration' [6], amount of substance concentration c_X, charge density ρ_{el}, mass density ρ_X, and volume fraction Φ_X of entity X in a total volume V are then expressed in consistent units of $[\mathrm{x/m^3}]$, $[\mathrm{mol/m^3}]$, $[\mathrm{C/m^3}]$, $[\mathrm{kg/m^3}]$, and $[\mathrm{m^3/m^3}]$, respectively. The quantity count depends on quantization U_X of entities X, with a minimum value of $N_X = 1\ \mathrm{x}$ (compare the questionable use of symbol 1: $N_X = 1\ 1$). However, if one accepts a *zero* count $N_X = 0\ \mathrm{x}$ without item to be counted (Figure 1), then *counting* numbers are equivalent to *natural* or *whole* numbers, represented by the numerals 0, 1, 2, 3, 4, 5, ..

A formally correct use of the following terms is not always easy: number, numeral (representing numbers in a specific numeral system, such as 4; VIII; or 9 053.78), and digit (or character; such as the ten characters in the decimal Arabic numeral system: 0, 1, 2, 3, .., 9). "Numbers may be divided in groups of three in order to facilitate reading; neither dots nor commas are ever inserted in the spaces between groups" ([1] p. 162). Here the term 'numbers' is used incorrectly and must be replaced by *digits* (IUPAC): "To facilitate the reading of long numbers the digits may be grouped in threes about the decimal sign but no point or comma should be used except for the decimal sign" [6].

In summary, a count should not be a dimensionless number, and the symbol for a unit should not be a numeral. The elementary unit x is the abstract representation of the entetic Euclidean Unit U_X. The count is a quantity that depends on the prior definition of the item to be counted. Without U_X, the dimension and unit of N_X would be undefined or semantically void.

5. Derived quantities containing a count

Counting and measuring are the two fundamental methods to quantify anything – any entity or sample. Counting and measuring may be considered as the most basic concepts in experimental science. The *size* of something is quantified by *measuring* extensive quantities such as mass m_S [kg] or volume V_S [m^3] of the sample S (Figure 2a). The *value* of a quantity Q_u is the product of a number N and an *abstract* unit u_Q. Similarly, the *value* of a count N_X is the product of a number N and the *abstract* unit x. A *count* N_X is the number of individual items U_X (Figure 2b). Thus counting is equivalent to enumeration [7]. A count of *events* results from monitoring repetitions of identical occurrences observed simultaneously or sequentially in time. Countable *objects* are discrete individual items in space, in contrast to continuous quantities that cannot be counted but are measured (Figure 2).

Elementary quantities that are stoichiometrically linked to counting of elementary entities are compared with extensive quantities based on measurements of a pure sample (Figure 2). Count, amount, and charge depend on the definition of elementary entities, whereas measurements of extensive quantities are independent of information on the quantized or continuous nature of the entity X. Entity X is used strictly in the sense of *entity-type* X. In practical language in the context of human and large animal bodies, however, the entity-type X = *body* is understood as indicating the elementary entity in terms of a single individual body, linked to the perception that body mass is measured routinely on a sample of a single individual organism.

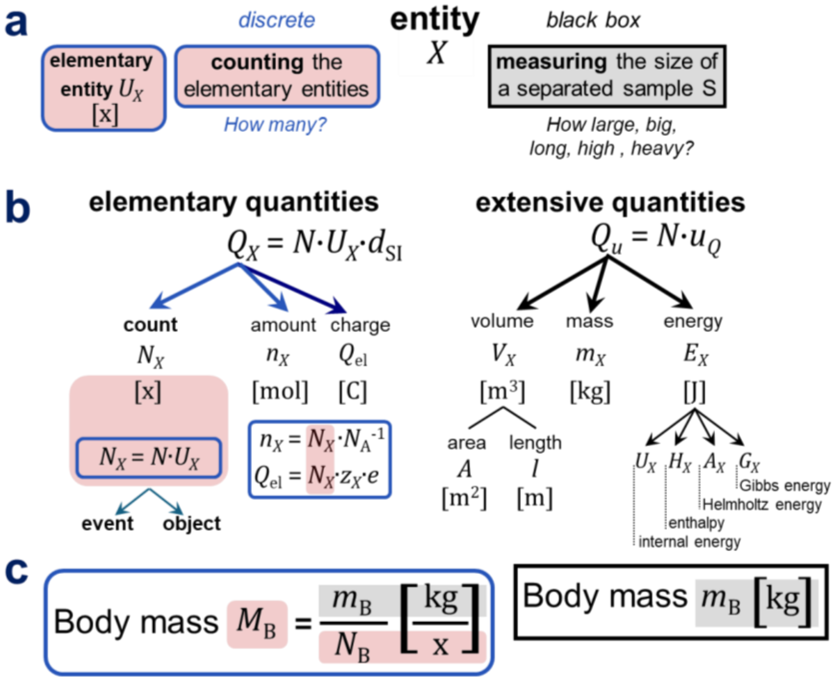

Figure 2. Elementary quantities linked to count (left) and extensive quantities based on measurements (right). (**a**) The elementary entity, the single individual occurrence of a discrete entity X defines the Euclidean Unit U_X. Measuring the size of a sample is a black-box approach without counting. (**b**) All elementary quantities Q_X are defined as a count N_X multiplied by a defining SI constant d_{SI}. d_{SI} = 1 for the count, d_{SI} = N_A^{-1} for amount, and $d_{SI} = z_X \cdot e$ for charge. Quantities Q_u are expressed in *abstract* units u_Q, applicable to any kind of sample of objects X. (**c**) Elementary mass M_X is the extensive quantity mass m_X normalized for the count N_X, illustrating the ambiguity of the term *body mass*.

Conceptual coherence is achieved by elevating the status of U_X [x] from an implicit notion to a formal element in quantity calculus. This facilitates dimensional analysis and prepares the ground for a consistent metrological treatment of counts and their associated elementary quantities. A distinction must be made between the mass m_X 'of' a sample of type X and the mass M_X 'per' entity U_X. Cell mass m_{ce} [kg] is the mass *of cells*, ce, in the sample analyzed on a mass balance. For this measurement, the number of cells N_{ce} [x] in the sample is irrelevant. The cell count N_{ce}, however, must be determined to express the mass *per cell*, $M_{ce} = m_{ce}/N_{ce}$ [kg/x].

When measuring the body mass m_B *of an individual*, the black box of a mass balance (Figure 2a) must be opened to obtain the additional information on the count N_B = 1 x. Then the elementary body mass or the mass *per individual* is $M_B = m_B/N_B$ [kg/x]. m_B may refer to an ensemble of items. Therefore, the elementary unit x does not imply that N_B = 1 x. An explicit distinction is required between the units [kg] and [kg/x] of the extensive and elementary quantities, m_X and M_X, respectively (Figure 2c).

Confusion with the symbol M for molar mass [kg/mol] [6] must be carefully avoided by using — if necessary — the more explicit symbol M_{U_X} for the elementary mass (Table 1).

Table 1. Count, amount, volume, mass normalized for N_X, n_X, V_X, and m_X. Canonical symbols in a consistent formal system. Compared with commonly used practical symbols, the canonical symbols help to decode the meaning of the quantities expressed by these symbols by showing their isomorphic form. For example, compare the Avogadro constant N_A and molar mass $M(X)$ with isomorphic canonical symbols N_{n_X} and M_{n_X}; or count concentration C_X, the inverse of the molar volume $V_m(X)^{-1}$, and density ρ_X with isomorphic canonical symbols N_{V_X}, n_{V_X}, and m_{V_X}, respectively.

Quantity	Unit		Normalized for quantity	Unit	=	Canonical symbol	Unit	Practical symbol	Quantity
count N_X	x	:	count N_X	x	=	1	1	-	-
amount n_X	mol	:	count N_X	x	=	n_{U_X}	mol/x	N_A^{-1}	1/Avogadro constant
volume V_X	m³	:	count N_X	x	=	V_{U_X}	m³/x		elementary volume
mass m_X	kg	:	count N_X	x	=	M_{U_X}	kg/x		elementary mass
count N_X	x	:	amount n_X	mol	=	N_{n_X}	x/mol	N_A	Avogadro constant
amount n_X	mol	:	amount n_X	mol	=	1	1	-	-
volume V_X	m³	:	amount n_X	mol	=	V_{n_X}	m³/mol	$V_m(X)$	molar volume (IUPAC)
mass m_X	kg	:	amount n_X	mol	=	M_{n_X}	kg/mol	$M(X)$	molar mass
count N_X	x	:	volume V_X	m³	=	N_{V_X}	x/m³	C_X	number concentration (IUPAC)
amount n_X	mol	:	volume V_X	m³	=	n_{V_X}	mol/m³	$V_m(X)^{-1}$	1/molar volume
volume V_X	m³	:	volume V_X	m³	=	1	1	-	-
mass m_X	kg	:	volume V_X	m³	=	m_{V_X}	kg/m³	ρ_X	density
count N_X	x	:	mass m_X	kg	=	N_{m_X}	x/kg		
amount n_X	mol	:	mass m_X	kg	=	n_{m_X}	mol/kg	$M(X)^{-1}$	1/molar mass
volume V_X	m³	:	mass m_X	kg	=	V_{m_X}	m³/kg	v_X	specific volume (IUPAC)
mass m_X	kg	:	mass m_X	kg	=	1	1	-	-

".. the masses of a silicon atom (averaged over the three isotopes used for the sphere) m_{Si}, and the electron m_e .." ([1] p. 205). Defining entity X as Si, the SI symbol for the mass of a sample of Si is m_{Si} [kg]. From a count N_{Si} of Si in the sample, the mass per silicon atom is m_{Si}/N_{Si} [kg/x]. The same holds for the mass per any elementary entity of entity-type X. For consistency, the general term 'mass of an entity' ("masses of a silicon atom .. and the electron") has to be replaced by the general term 'mass per elementary entity' (masses *per* silicon atom .. and *per* electron). The symbol m_{Si} cannot be used for the expression m_{Si}/N_{Si}. This formal inconsistency must be resolved in the SI to achieve coherence. Elementary mass per Si atom is $M_{U_{Si}} = m_{Si}/(N \cdot U_{Si})$ [kg/x].

The symbol M(X) ([1] p. 210) indicates the molar mass [kg/mol] of entity X. There remains a problem with an extension to obtain a consistent system of symbols for other derived quantities, when mass is normalized not only for amount [kg/mol] but count [kg/x] or volume [kg/m^3]. In different contexts there are *ad hoc* practical symbols in use, but a formally consistent system of symbols does not exist, as shown in Table 1, comparing canonical and practical symbols for harmonization.

Count N_X, amount n_X, and charge Q_{el} are the three elementary quantities Q_X of countable, discrete entities which are frequently obtained by conversion of primary measurements rather than by direct application of various counters. In any case, the value of an elementary quantity Q_X is a count N_X multiplied by a defining SI constant d_{SI} (Figure 2b). Elementary and continuous extensive quantities are distinguished as $Q_X = N \cdot U_X \cdot d_{SI}$ and $Q_u = N \cdot u_Q \cdot$(Figure 2b). Subscripts X and Q for 'entity-type' and 'quantity-type', respectively, point to the contrasting meanings of the two fundamental definitions of an *abstract* versus *entetic* 'unit'. Comparable to *amount* of substance, the proper name for 'quantity for counting entities' is *count*. It is not a quantity for entities that are counting, such as counting machines, cell counters, or ticket counters.

6. Discussion

Concerns about inconsistencies in the SI due to the omission or ambiguous interpretation of the unit "one" for counting quantities are not new [3]. Proposals to introduce distinct unit symbols for number of molecules [mcl], number of atoms [atm], or number of particles [pcl] and to adopt ‘ent’ as a general unit symbol for ‘entity’ [13] risk conflating the fundamentally different concepts of *abstract* units in contrast to a *realized* elementary entity U_X. The abstract unit [x] is intended to denote ‘one instance of a countable entity’ in a general, unit-consistent manner, whereas a realized elementary entity U_X is a representative example defined in relation to the specific context of enumeration. Accordingly, quantities expressed in units ‘1’, ‘ent’, and — likewise — ‘x’ require explicit specification of the actual entity type in question to convey meaningful information.

For example, the rate of O_2 consumption per cell (I_{O2}) is not appropriately expressed as amol $O_2 \cdot s^{-1}$/cell, since ‘O_2’ is specified in the context of the quantity I_{O2} and ‘cell’ is not an SI unit. Rather, I_{O2} should be expressed in the unit [amol·s^{-1}/x], where [x] represents the abstract unit of count [11]. The specific nature of the entities involved — O_2 molecules and cells — is defined as part of the description of the quantity itself, not embedded within the unit. The abstract units mol and x never tell — and are not supposed to tell — the story about the entetic identity.

According to Mohr and Phillips [13], the SI-definition of hertz as $\mathrm{Hz} = s^{-1}$ should be reconsidered in favor of a more explicit formulation, such as $1\ \mathrm{Hz} = 1\ \mathrm{cnt \cdot s^{-1}} = 1\ \mathrm{cyl \cdot s^{-1}} = 2\ \pi \cdot \mathrm{rad \cdot s^{-1}}$. However, this reformulation risks conflating abstract units with realized entities: counts, cycles, and 2π radians are not abstract units but refer to specific types of elementary entities, either events or objects (Figure 2). As emphasized by Brown and co-authors [2, 4], such descriptions belong to the definition of the quantity, not the SI unit. Frequency, in its general form, is defined as the number of specified events per unit time. To be countable, an event must be identifiable as an elementary entity U_X, associated with the abstract unit [x] (Figure 2). Accordingly, the definition of the unit hertz should be generalized as $\mathrm{Hz} \stackrel{\mathrm{def}}{=} \mathrm{x}/s$, where [x] denotes the abstract unit of the specified instance of counted events. This conforms with the guiding principle of the SI that units "are never qualified by further information about the nature of the quantity; any extra information on the nature of the quantity should be attached to the quantity symbol and not to the unit symbol" ([1] p. 149). The periodic or stochastic character of an event — such as whether it is a cycle or a radioactive decay — should therefore be described explicitly in the quantity's definition, not encoded in the unit. Expressing frequency using the generalized unit $\mathrm{x} \cdot s^{-1}$ shifts the specification of entity-type to the level of the Euclidean Unit U_X, leaving [x] as the consistent, abstract count unit. "The SI unit of activity is becquerel, implying counts per second" ([1] p. 140). By adopting the elementary unit x for the quantity count, the formal structure of the SI becomes more unified and transparent, potentially rendering separate unit names such as becquerel and $\mathrm{rad \cdot s^{-1}}$ unnecessary.

Table 2. Elementary entity and five defining SI constants [1] with units containing the elementary unit, with explicit symbol x, comparing explicit units and SI units.

Name	**Symbol**	**Numerical value**	**Explicit unit**	**SI unit**
elementary entity	U_X	1	x	1
hyperfine transition frequency of Cs	$\Delta\nu_{Cs}$	9 192 631 770	$\mathrm{x \cdot s^{-1}}$	Hz; $\mathrm{s^{-1}}$
Avogadro constant	N_A	$6.022\ 140\ 76 \cdot 10^{23}$	$\mathrm{x \cdot mol^{-1}}$	$\mathrm{mol^{-1}}$
Planck constant	h	$6.626\ 070\ 15 \cdot 10^{-34}$	$\mathrm{J \cdot s \cdot x^{-1}}$	$\mathrm{J \cdot s}$
Boltzmann constant	k	$1.380\ 649 \cdot 10^{-23}$	$\mathrm{J \cdot K^{-1} \cdot x^{-1}}$	$\mathrm{J \cdot K^{-1}}$
elementary charge	e	$1.602\ 176\ 634 \cdot 10^{-19}$	$\mathrm{C \cdot x^{-1}}$	C

Five of the seven defining SI constants contain the unit x hidden in the SI units (Table 2). The unit x cancels in several important quantities that are products or ratios of these constants, such as the product of the Planck constant and frequency of radiation, $h{\cdot}\nu$, with units [$\mathrm{J{\cdot}s{\cdot}x^{-1}}$] and [$\mathrm{x{\cdot}s^{-1}}$], respectively, the 'universal frequency' kT/h [$\mathrm{x{\cdot}s^{-1}}$], the molar gas constant $R = k{\cdot}N_A$ [$\mathrm{J{\cdot}K^{-1}{\cdot}mol^{-1}}$], or the electromotive constant $f = k{\cdot}e^{-1}$ [$\mathrm{J{\cdot}K^{-1}{\cdot}C^{-1}}$] (Table 2) [10].

The notion "'Metre' is to the concept 'length' as '1' is to the concept 'number'" [2] is extended by the realization that — opposite to the concept of 'number' — 'x' is related to the concept of 'count'. Abandoning the unit symbol 1 in favour of the symbol x for the count can be understood in historical context. McGlashan [12] rejected "the argument that the mole should be regarded as having the dimension one (or number)" [2]. Just as the unit symbol 'mol' (and not 'number of moles') is used to denote the quantity amount of substance, so too should the symbol 'x' (and not 'number of ones') be employed to represent the quantity count with full dimensional status. The symmetry between mol and x — heralded in the historic "new approach to articulating the definitions of the units in general" ([1] p. 122) — emphasizes that count, like amount, is not dimensionless and deserves explicit representation in scientific notation.

The ambivalent term 'unit' with its dual meanings is used and confused in practical language and the scientific literature. In the elementary entity U_X, the unit (the 'one') relates to the entity-type X, to the single individual thing (individual or undivided; the root of the word *thing* has the meaning of 'assembly'). In contrast to counting, a unit u_Q is linked to the measurement of quantities $Q_u = N{\cdot}u_Q$, such as volume, mass, energy. These quantities — and hence the units u_Q — are abstracted from entity-types, pulled away from entetic things. This should resolve the confusion regarding the distinction between entities and units [7]. The new SI has completed the total abstraction of units, from the previous necessity to not only provide a quantitative definition but also a physical realization of a unit in the form of an 'artefact', such as the International Prototype of the Kilogram. The new definitions of the base SI units are independent of any physical realization: u_Q is separate from X. In agreement, the unit x is separate from the nature of countable entities U_X.

7. Conclusions

The quantity *count* N_X merits an as yet insufficiently recognized fundamental role in the SI. A consistent symbol for the quantity *count* needs to be defined (Table 1). A unit and appropriate symbol for the unit of the quantity count must not be confused with a number, and a numeral is not an appropriate symbol for a unit. The term 'unit one' is a tautology, since 'unit' and 'one' contain essentially the same message. The nature of 'a single individual thing' is associated with the elementary entity or Euclidean Unit U_X, expressed in the abstract unit x. Deficiencies in the current SI Brochure of a proper treatment of the quantity count should be taken into account by implementation of a Consultative Committee on 'Elementary unit and count'.

Abbreviations: e: elementary charge; m_X: mass of a sample of X; M_{U_X}: elementary mass, per elementary entity; n_X: amount of X; N: pure number; N_A: Avogadro constant;

N_X: count of a sample of X; Q_u: quantity expressed in abstract units; Q_X: elementary quantity linked to a count; SI: International System of Units; u_Q: abstract unit of quantity Q_u; U_X: elementary entity; V_{UX}: elementary volume, per elementary entity; V_X: volume of a sample of X; x: elementary unit; X: entity type

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