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Theoretical Communication

Boltzmann and gas constant – ambiguities between energy and force

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Summary

Energy and force are fundamental quantities, differing in both their physical meaning and units. In the International System of Units (SI) Brochure, the Boltzmann constant is one of the seven defining constants. The Bureau International des Poids et Mesures explains the Boltzmann constant k as “a proportionality constant between the quantities temperature (with the unit kelvin) and energy (with the unit joule)”, and consequently, “one kelvin is equal to the change of thermodynamic temperature that results in a change of thermal energy kT by 1.380 649 × 10^{-23} J” (SI Brochure 2019, 9th edition, version 3.01, August 2024). It should be noted that this groundbreaking definition concerns a physical constant, not the concept of energy itself. But a clear distinction is warranted: Energy in any form and expressed in the unit joule is an *extensive* quantity that scales in direct proportion to the size of a system, whereas *intensive* quantities such as temperature and force are independent of system size. The concept that a fixed temperature change results in a proportional change of thermal energy kT lacks specification of the entity to which energy is attributed. By relating energy to the individual particle or event, kT accounts for the quantum structure of nature. The present analysis aims at resolving ambiguities of the term ‘thermal energy kT’ by a focus on SI units and corresponding symbols to describe adequately and clearly the full meaning of any form of energy related to fundamental physical or thermodynamic forces.

Cite
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$$\begin{tikzcd}
 & kT \left[\frac{\mathrm{J}}{\mathrm{x}}\right] \arrow[dl, leftrightarrow, "N_\mathrm{A}"'] \arrow[dr, leftrightarrow, "e", "z_\mathrm{Y}"'] & \\
\left[\frac{\mathrm{J}}{\mathrm{mol}}\right] RT \arrow[rr, leftrightarrow, "z_\mathrm{Y}", "eN_\mathrm{A}"'] & & fT\cdot\frac{1}{z_\mathrm{Y}}\left[\frac{\mathrm{J}}{\mathrm{C}}\right]
\end{tikzcd}$$

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Faraday constant F
force
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ion
motive force

Significance

Energy and force are foundational yet often conflated concepts in thermodynamics. While heat and temperature are clearly distinguished, the product of the Boltzmann constant and temperature (kT) is commonly misinterpreted as “thermal energy” rather than recognized as a *motive force quantum* — an intensive quantity distinct from any form of energy. This work clarifies the dimensional roles of fundamental constants by emphasizing the importance of explicit units, resolving persistent ambiguities in expressing energy versus force. By bridging macroscopic and microscopic thermodynamics, the proposed framework enhances conceptual rigor, supports dimensional consistency, and improves communication across physics, chemistry, and systems biology.

1. Introduction

On 20 May 2019 new definitions of several constants defining the SI units entered ‘into force’ [1]. Notably, while we say that legal or scientific definitions ‘enter into force’, it would be linguistically and conceptually inappropriate to state they ‘enter into energy’. In everyday language, we tend to discriminate intuitively between energy and force. Scientific terminology, however, has historically blurred the two. For two centuries following Newton’s *Principia* (1687), which formalized mechanical and gravitational force, the conceptual boundary between force and energy remained imprecise. Indeed, what is now recognized as the first law of thermodynamics — conservation of internal energy — was originally formulated as the principle of conservation of *force* [2, 3]. The fundamental forces of physics are the gravitational, electroweak (combining electromagnetic and weak nuclear) and strong nuclear forces. These forces are vectors, directed parallel to a spatial gradient, and are effective at a distance. In contrast, the electromotive force emF defined by the Nernst equation is a difference of electrochemical potentials. Direction of a gradient in space of physical forces is replaced in the emF by a potential difference between separate electrodes or compartments (the anode and cathode). Energy per distance l (per meter) of physical forces, with unit newton $[\mathrm{N} = \mathrm{J}/\mathrm{m}]$, is replaced in the emF by energy per charge Q_{el} (per coulomb), with unit volt $[\mathrm{V} = \mathrm{J}/\mathrm{C}]$. Both units N and V express ratios of ‘free’ energy (related to work) to a ‘motive’ quantity, making the fundamental physical and electromotive forces comparable as isomorphic *motive forces* [4]. These are known as generalized forces in the thermodynamics of irreversible processes [5-8]. The present perspective considers the fundamental difference between ‘thermal energy kT’ and thermodynamic motive forces. kT takes us from the macroscopic scale of molar quantities to the microscopic or quantum scale of Schrödinger’s wave mechanics and molecular or particle formats, where even more strict definitions are required for proper analysis [9]. Note that Erwin Schrödinger explicitly emphasized dimensional consistency in his formulation of quantum mechanics, defining the dimension of K in his Eq. 9 [10] as action – the product of energy and time. Implicit in this definition is that K, and therefore the Planck constant h, represent action *per elementary entity* – whether a particle or an event.

2. The units of the Boltzmann constant and the gas constant

The Boltzmann constant k, Avogadro constant N_A, and elementary charge e are among the seven SI constants redefined in 2019 as numerically fixed values [1]. These three constants share a common connection: each is fundamentally related to "the quantity number of elementary entities (with the unit one, symbol 1)" [1]. However, in the SI definitions, the symbol 1 is typically omitted — obscuring the fact that k and e are quantities expressed *per entity* whereas N_A is the number *of entities* per amount.

To make this dimension explicit, the symbol x is introduced to represent the unit of countable entities [4, 11]. Instead of the SI symbol 1, the explicit symbol x for the count serves a conceptual purpose: it highlights the distinction between energy and thermodynamic force — namely, presenting energy per *count* $[\mathrm{x}]$ analogous to energy per *amount* $[\mathrm{mol}]$ or per *charge* $[\mathrm{C}]$. Expressing the Boltzmann constant in the unit $[\mathrm{J{\cdot}K^{-1}/x}]$ clarifies that kT represents energy per entity, whereas omitting the symbol 1 (as in $\mathrm{J{\cdot}K^{-1}}$) conceals this interpretation [11, 12]. Formally, this distinction becomes evident in the ideal gas law, where the energy associated with pressure-volume work can be expressed in two distinct but representationally isomorphic formats: one referring to energy per amount of substance n_B (amount of ideal gas B; Eq. 1a), the other per elementary entity N_B (the corresponding count, Eq. 1b):

$$\text{molar format:} \qquad RT = \frac{pV}{n_\mathrm{B}} \qquad [\mathrm{J/mol}] \tag{1a}$$

$$\text{molecular format:} \qquad kT = \frac{pV}{N_\mathrm{B}} \qquad [\mathrm{J/x}] \tag{1b}$$

These expressions are structurally identical in form and meaning, yet differ in their referential framework: In both cases, pressure-volume work pV $[\mathrm{J}]$ is divided by an extensive quantity — amount or count — to yield a motive force. Expressing kT in the explicit unit $[\mathrm{J/x}]$ helps clarify that it is not energy, but energy *per elementary entity* — a thermodynamic driving force or, more precisely, a motive force quantum accounting for the quantum structure of nature (Figure 1) [4].

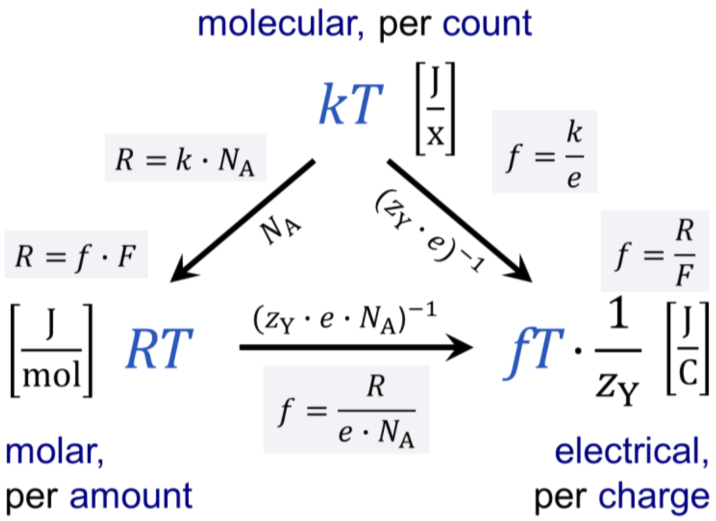

Figure 1. Motive force quantum in three formats which may be conceived as the ‘Boltzmann plane’: kT (per count), RT (per amount), and fT/z_Y (per charge of Y) corresponding to the Boltzmann constant, gas constant, and electromotive constant, respectively (Table 1). The three isomorphic units are shown in brackets. The unit of the quantity count is represented by the symbol x. Modified from [4].

Table 1. Motive force quantum defined by SI constants (k, N_A, and e) and expressed in different formats of the Boltzmann constant k, gas constant R, and electromotive constant f. Values of these quantum effects are shown for the standard temperature of 25 °C ($T° = 298.15$ K). The motive force quantum at any temperature T is obtained by multiplication by $T/T°$. Multiplication factors for 0 °C and 37 °C are 0.916 and 1.040, respectively; $\ln(10) \approx 2.3026$.

Format	**Relation**	**Motive force quantum** times $\ln(10)$	**Unit**
particle	$kT = RT \cdot \frac{1}{N_A} = fT \cdot e$	$kT° \cdot \ln(10) = 9.4784 \cdot 10^{-21}$	J/x
chemical	$RT = kT \cdot N_A = fT \cdot F$	$RT° \cdot \ln(10) = 5.7080 \cdot 10^{3}$	J/mol
electrical	$fT = kT \cdot \frac{1}{e} = RT \cdot \frac{1}{F}$	$fT° \cdot \ln(10) = 59.159 \cdot 10^{-3}$	J/C

3. Gibbs force

In physical chemistry, it is conventional to normalize extensive quantities by the amount of substance and use the same names and symbols for both the extensive and the resulting intensive quantities [13]. This practice, while convenient in context, fosters ambiguity when the intensive terms are discussed without explicit reference to their normalized basis. A prominent example is the ‘reaction Gibbs energy’ $\Delta_r G$ with the unit [J/mol], as defined by the International Union of Pure and Applied Chemistry (IUPAC) [13]. Although this quantity is intensive and functions as a thermodynamic driving force, it is still termed ‘energy’ rather than ‘force’. To improve conceptual clarity, this intensive term should be redefined as the Gibbs *force* of reaction $\Delta_r F$ [J/mol], distinct from the extensive quantity Gibbs energy change $\Delta_r G$ [J] [4, 8]. Analogously, the widely used term 'thermal energy' is problematic in relation to kT expressed in joules $[\mathrm{J}]$ – masking its role as an intensive motive force per elementary entity $[\mathrm{J/x}]$ rather than as total energy (Figure 1). Clarifying this distinction is essential for accurately characterizing the driving forces in thermodynamics and chemistry.

In contrast to this chemical tradition, electrochemistry provides a rigorous terminological separation between energy and force. When ‘Gibbs reaction energy’ is calculated as the product of a redox potential difference $[\mathrm{J{\cdot}C^{-1}}]$ and $n{\cdot}F$ $[\mathrm{C}]$, the result is total energy $[\mathrm{J}]$, not energy per mole $[\mathrm{J{\cdot}mol^{-1}}]$ or per entity $[\mathrm{J{\cdot}x^{-1}}]$ [14]. In this important publication, the quantity n is defined as the ‘number of electrons’, but in context the count $[\mathrm{x}]$ must be corrected to amount $[\mathrm{mol}]$ of electrons for dimensional coherence. Electrochemistry reinforces the distinction between energy and force through the explicit unit volt $[\mathrm{V{=}J{\cdot}C^{-1}}]$, which parallels the newton $[\mathrm{N = J{\cdot}m^{-1}}]$. In contrast, no specific unit names and symbols exist for thermodynamic forces per amount or per extent of reaction $[?? = \mathrm{J{\cdot}mol^{-1}}]$ and per count or extent of elementary advancement $[?? = \mathrm{J{\cdot}x^{-1}}]$ in chemistry and statistical mechanics. This lack of terminological and iconic differentiation contributes to the persistent ambiguity between (potential) energy and driving force (electrochemical potential) in chemical thermodynamics. Introducing consistent unit notation, primarily differentiating between $[\mathrm{J{\cdot}x^{-1}}]$ and $[\mathrm{J}]$, provides the necessary precision to align molecular chemistry with the clarity long established in electrochemistry. Beyond the stringent distinction between forms, qualities, or simply sets of energy (‘thermal energy’, kinetic and potential energy, work and heat, Helmholtz and internal energy, Gibbs energy and enthalpy) [3], this improved framework supports the growing demands of quantum chemistry and quantum biology, where thermodynamic reasoning must be reconciled with processes occurring at the level of charged particles and molecular entities.

4. The electromotive constant in electrochemistry

> ‘The phenomena associated with electrolysis were studied by M. Faraday (1832-33), and the nomenclature which he used, and which is still employed, was devised for him by W. Whewell. .. Faraday assumed the flow of electricity to be associated with the movement of charged particles ..; these were called *ions* (Greek: wanderer).’ – Samuel Glasstone (1948) [15]

Charged motive particles are properly called ions — a term introduced through the collaboration of Faraday and Whewell to describe entities that *wander* under the influence of an electric potential difference [16]. This etymology resonates with the concept of *motive force*, which drives ion translocation across membranes or electrode systems (Table 1). The Nernst equation quantifies the resulting electromotive force (emF) for a one-component system, $emF \overset{\text{def}}{=} \Delta\Psi_{\mathrm{Y}}$, as:

$$emF = emF^{\circ} + \frac{RT}{F}\cdot\frac{1}{z_{\mathrm{Y}}}\cdot\ln\left(\frac{a_{\mathrm{Y}_b}}{a_{\mathrm{Y}_a}}\right) \qquad [\mathrm{J/C}] \qquad (2)$$

where z_Y is the charge number of ion Y, and a_{Y_a} and a_{Y_b} are its activities in compartments a and b ($a \rightarrow b$). The standard electromotive force is $emF^\circ = 0\ \mathrm{V}$ at identical activities in both compartments ($a_{\mathrm{Y}_a}{}^\circ = a_{\mathrm{Y}_b}{}^\circ$). $F = e \cdot N_\mathrm{A}$ $[\mathrm{C/mol}]$ is the Faraday constant. In this context, the *electromotive constant* f is introduced as $f = R/F$, combining the Boltzmann constant and elementary charge ($f = k/e$), analogous to the combination of the Boltzmann constant and Avogadro constant in the gas constant ($R = k \cdot N_\mathrm{A}$; Table 2).

In summary, three constants multiplied by T express the motive force quantum in their respective units: kT per elementary unit $[\mathrm{x}]$, RT per mole $[\mathrm{mol}]$, and fT per coulomb $[\mathrm{C}]$ (Tables 1 and 2; Figure 1). Notably, the ratio RT/F (Eq. 2) appears in 59 equations in the chapter *Electrochemistry* (Glasstone 1948; [15]). More recently, the terms RT/F or kT/e occur in 24 equations across five chapters of *Biothermodynamics* [17]. Avoiding the detour through these ratios by introducing the electromotive constant f simplifies both the presentation and interpretation of equations, which is of particular relevance in an interdisciplinary context (see Eq. 3 and Eq. 5 below).

To align Eq.(2) with logarithmic activity measures of ion-selective electrodes commonly used in chemistry and biology (e.g., pH), the natural logarithm is replaced with a base-10 logarithm, yielding Eq.(3) as a base-10 version of Eq.(2):

$$emF = -fT \cdot \frac{1}{z_\mathrm{Y}} \cdot \ln(10) \cdot \Delta \mathrm{pY} \qquad [\mathrm{J/C}] \qquad (3)$$

where $\mathrm{pY} = -\log_{10}(a_\mathrm{Y})$, such that $-\Delta\mathrm{pY}$ corresponds to the compartmental direction of ion flow ($a \rightarrow b$). It is practical to multiply the motive force quantum by $\ln(10)$ (Table 1).

Ultimately, using the symbol x for the SI unit one (Table 2) makes explicit the physical meaning of energy per elementary entity. It helps differentiating between the extensive thermodynamic quantity energy $[\mathrm{J}]$ and intensive motive force $[\mathrm{J/x}]$, improving conceptual clarity in the interpretation of fundamental thermodynamic equations such as the ideal gas law and the Nernst equation.

Table 2. Three defining SI constants (k, N_A, and e) [1] with units containing the explicit symbol x [11]. The elementary unit x cancels in the derived constants F, R, and f.

Constant	Relation			Numerical value	Unit	
Boltzmann constant	k	$= f \cdot e$	$= R/N_\mathrm{A}$	$= 1.380\ 649 \cdot 10^{-23}$	$\mathrm{J \cdot K^{-1}}$	/x
Avogadro constant	N_A	$= R/k$	$= F/e$	$= 6.022\ 140\ 76 \cdot 10^{23}$	x	/mol
elementary charge	e	$= k/f$	$= F/N_\mathrm{A}$	$= 1.602\ 176\ 634 \cdot 10^{-19}$	C	/x
Faraday constant	F	$= e \cdot N_\mathrm{A}$	$= R/f$	$= 96\ 485.332\ 12$	C	/mol
gas constant	R	$= k \cdot N_\mathrm{A}$	$= f \cdot F$	$= 8.314\ 462\ 618$	$\mathrm{J \cdot K^{-1}}$	/mol
electromotive constant	f	$= k/e$	$= R/F$	$= 8.617\ 333\ 262 \cdot 10^{-5}$	$\mathrm{J \cdot K^{-1}}$	/C

A motive force of fundamental importance in bioenergetics is the protonmotive force pmF which drives ATP synthesis in mitochondria, bacteria, and chloroplasts by a chemiosmotic process [18, 19]. This force is generated in mitochondria by the action of molecular proton pumps that translocate protons (H^+) from the negatively charged matrix (cathodic compartment) across the mitochondrial inner membrane to the positively charged intermembrane space (anodic compartment). The pmF is the sum of two partial motive forces: (*1*) The electromotive contribution pmF_el results from the distribution of all ions generating an electric potential difference according to Eq.(3). (*2*) Irrespective of charge, the partial diffusional contribution pmF_d is related to the pH difference $\Delta\mathrm{pH}$ across the semipermeable inner membrane. The diffusional pmF_d is the partial Gibbs energy change $[\mathrm{J}]$ per advancement of proton translocation, where advancement [7] can be expressed per H^+ pumped out of the cathodic compartment, with unit $[\mathrm{J/x}]$ (Eq. 4a) or $[\mathrm{J/mol}]$ (Eq. 4b). Alternatively, the diffusional pmF_d can be expressed in electrical units of volts $[\mathrm{J/C}]$ (Eq. 4c). Unlike Eq.(3), Eq.(4) omits the charge number z_{H^+} because the diffusional component pmF_d reflects a statistical (entropic) force arising from particle distribution. It is additive to the electric component pmF_el, yet independent of the ion's charge [4, 19, 20].

$$\text{molecular format:}\qquad pmF_\mathrm{d}(\mathrm{count}) = -kT\cdot\ln(10)\cdot\Delta\mathrm{pH}\qquad [\mathrm{J/x}]\qquad (4a)$$

$$\text{molar format:}\qquad pmF_\mathrm{d}(\mathrm{amount}) = -RT\cdot\ln(10)\cdot\Delta\mathrm{pH}\qquad [\mathrm{J/mol}]\qquad (4b)$$

$$\text{electrical format:}\qquad pmF_\mathrm{d}(\mathrm{charge}) = -fT\cdot\ln(10)\cdot\Delta\mathrm{pH}\qquad [\mathrm{J/C}]\qquad (4c)$$

Boltzmann, a pioneer of statistical thermodynamics, laid the foundation for describing diffusion at the level of individual particles. In particle physics and quantum biology, diffusion is analyzed in the molecular or particle format, as captured in Eq.(4a) [21]. In the chemical and electrical formats, the Boltzmann constant is replaced by the gas constant R (Eq. 4b) and the electromotive constant f (Eq. 4c), respectively.

Another example for the relevance of the electromotive constant f comes from electrostatics and the Debye-Hückel theory. The local charge density of a particular ion $[\mathrm{C/m^3}]$ at a given distance from the central ion is related to the local electric potential of the perturbed field $[\mathrm{V{=}J/C}]$ and ionic strength $[\mathrm{mol/m^3}]$, with a proportionality coefficient $\frac{N_\mathrm{A}e^2}{kT}$ (Eq. 4.31 in [22]). This term is difficult to grasp. Introducing $N_\mathrm{A}\cdot e = F$ and $k/e = f$ (Table 2), this proportionality coefficient is,

$$\frac{N_\mathrm{A}\cdot e^2}{kT} = \frac{F}{fT}\qquad \left[\frac{\mathrm{C}^2}{\mathrm{J\cdot mol}}\right]\qquad (5)$$

The Faraday constant F $[\mathrm{C/mol}]$ is required when the concentration term in ionic strength is expressed as amount concentration c_i $[\mathrm{mol/m^3}]$. However, when expressed as count concentration C_i $[\mathrm{x/m^3}]$, elementary charge e $[\mathrm{C/x}]$ replaces F. Consequently, the proportionality coefficient becomes $e\cdot(fT)^{-1}$. Moreover, when the local electric potential of the perturbed field is not expressed as a voltage but as energy per particle $[\mathrm{J/x}]$, the proportionality coefficient is reformulated as $e\cdot(kT)^{-1}$. Finally, if the local charge density of the ion is also expressed as a local count concentration $[\mathrm{x/m^3}]$, the unit conversion factor $N_\mathrm{A}\cdot e^2$ cancels from Eq.(5). The complex original proportionality coefficient in Eq.(5) can thus be decomposed into two conceptually distinguished steps: (*1*) expression of the motive force quantum in context-dependent units (Table 1). (*2*) unit conversion of the electrical potential and concentrations (Table 2). Taken together, the isomorphic proportionality coefficients unfold stepwise,

$$\frac{F}{fT}\left[\frac{\mathrm{C^2}}{\mathrm{J{\cdot}mol}}\right] \rightarrow \frac{e}{fT}\left[\frac{\mathrm{C/x}}{\mathrm{J/C}}\right] \rightarrow \frac{e}{kT}\left[\frac{\mathrm{C}}{\mathrm{J}}\right] \rightarrow \frac{1}{kT}\left[\frac{1}{\mathrm{J/x}}\right]$$

This illustrates the utility of introducing the electromotive constant f and unit symbol x in quantity calculus.

5. Discussion

> ‘The only thing that happens is that an incident light wave of ever so small an amplitude will rise the force vibration of the system to a finite amplitude.’ – Erwin Schrödinger (1928) [10].

The ‘thing’ in the isomorphic Boltzmann constant k, gas constant R, and electromotive constant f presents a compelling case for reinterpreting the product kT not as ‘thermal energy’ but as the *motive force quantum* normalized for the ‘thing’ (Figure 1). This shift clarifies that k is not a proportionality constant between temperature [K] and energy [J], but more precisely, between temperature [K] and thermodynamic force [$\mathrm{J{\cdot}x^{-1}}$]. At the quantum level, kT replaces R and f by a generalized focus on the particle and molecular microscopic scale, where ‘the Boltzmann constant connects the entropy S with the number Ω of quantum-mechanically accessible states’ [1]. This perspective becomes increasingly relevant as thermodynamics and quantum biology extend to the molecular and single-photon domains [9, 14, 23-28].

In molecular biology, macroscopic ergodic assumptions underlying concepts such as heat and temperature (Boltzmann’s holode [29]) are applied when replacing electrochemical sensors (in the domain of fT) by molecular and ionic fluorescent probes in the domain of kT [30-33]. Therefore, concepts like ‘thermal energy’ have to be critically assessed. In this context, redefining kT as a motive force quantum provides conceptual coherence and dimensional precision in describing microscopic thermodynamic driving forces. This complements efforts towards standardization of sensors in quantum biology and ‘establishing a common lexicon’ [34].

Beyond specific applications, the present analysis reveals a broader metrological and educational opportunity. The redefinition of SI base units has reinforced the role of seven defining constants in metrology [1]. Five defining constants are linked to elementary entities: the hyperfine transition frequency of Cs $\Delta\nu_{\mathrm{Cs}}$ [$\mathrm{x{\cdot}s^{-1}}$], the Avogadro constant N_{A} [$\mathrm{x{\cdot}mol^{-1}}$], the Planck constant h [$\mathrm{J{\cdot}s{\cdot}x^{-1}}$], the Boltzmann constant k [$\mathrm{J{\cdot}K^{-1}{\cdot}x^{-1}}$], and the elementary charge e [$\mathrm{C{\cdot}x^{-1}}$]. A consistent dimensional treatment of these quantities strengthens theoretical clarity across disciplines [11]. Notably, the symbol x is not part of the SI, but is proposed as a notational aid to explicitly represent the abstract unit ‘one’ [12, 35] associated with the quantity count — the number of elementary entities [11]. While the SI permits omission of the symbol 1, its explicit inclusion [12] can be problematic. For instance, stating the number of molecules B as $N_\mathrm{B} = 7{\cdot}10^9\ 1$ is formally correct but visually confusing. Neither '$7{\cdot}10^9{\cdot}1$' nor '$7{\cdot}10^9$ times 1' resolve the issue without introducing formal inconsistency, analogous to avoiding '25 times $^\circ\mathrm{C}$'. In contrast, the format $N_\mathrm{B} = 7{\cdot}10^9\ \mathrm{x} = 7\ \mathrm{Gx}$ is consistent with SI unit structure and the use of SI prefixes (G = 10^9). Importantly, this allows seamless integration with derived units [11]. While omitting the symbol 1 (the numeral representing the number one) does not affect the mathematical validity of equations involving multiplication or division by 1, it obscures the conceptual distinction between energy $[\mathrm{J}]$ and force $[\mathrm{J/x}]$.

A rigorous distinction between extensive and intensive quantities resolves existing ambiguities of energy and force in thermodynamics and statistical mechanics. More broadly, the effort towards more concise scientific communication contributes to addressing the ambiguity crisis, complementary to the reproducibility crisis [36, 37]. Conceptual rigor is crucial for proper application of core thermodynamic principles in electrochemistry and quantum biology, helping to resolve persistent misunderstandings of the fundamental concept of the protonmotive force and linking thermodynamics and kinetics [4]. Clarifying the distinction between all forms of energy and motive forces will enhance interdisciplinary scientific communication and improve education. As the role of thermodynamics continues to expand in the analysis of complex biological and nanoscale systems, renewed attention to dimensional consistency and unit clarity will be essential to bridge theory, measurement, and application.

Terms and symbols (see also Table 2)

a_B	activity of B
C_B	count concentration of B $[\mathrm{x/m^3}]$
c_B	amount concentration of B $[\mathrm{mol/m^3}]$
emF	electromotive force
N_B	count of B $[\mathrm{x}]$
n_B	amount of B $[\mathrm{mol}]$
p	pressure $[\mathrm{Pa}]$
pmF	protonmotive force
$Q_{\mathrm{H^+}}$	charge of $\mathrm{H^+}$ $[\mathrm{C}]$
V	volume $[\mathrm{m^3}]$
z_Y	charge number of ion Y

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